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CP measurement in quantum teleportation of neutral mesons

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Abstract. Quantum teleportation using neutral pseudoscalar mesons shows novel connections between particle physics and quantum information. The projection basis, which is crucial in the teleportation process, is determined by the conservation laws of particle physics and is different from the Bell basis, as in the usual case. Here we show that one can verify the teleportation process by CP measurement. This method significantly simplifies the high energy quantum teleportation protocol. Especially, it is rigorous and independent of whether CP is violated in weak decays. This method can also be applied to general verification of Einstein–Podolsky–Rosen correlations in particle physics.

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1 Introduction

It has been suggested that a $K^0\bar{K}^0$ pair in an entangled state can be produced from the strong decay of a ϕ meson generated in electron–positron annihilation, or from a proton–antiproton collision, and that a similarly entangled $B^0\bar{B}^0$ pair can be produced from the $\Upsilon(4S)$ resonance [1–4]. Experimental results consistent with the existence of entanglement has been demonstrated in $K^0\bar{K}^0$ pairs produced in proton–antiproton annihilation in the CPLEAR detector in CERN [5], in $K^0\bar{K}^0$ pairs produced in ϕ decay in the KLOE detector in DAΦNE [6, 7], as well as in $B^0\bar{B}^0$ pairs produced in the BELLE detector in the KEKB electron–positron collider [8, 9]. Recently, the proposal was made to use neutral kaons to realize such quantum information processes as quantum teleportation and entanglement swapping in the regime of high energy physics [10]. In this proposal, the process is verified by measuring the strangeness ratio of the particle to which the state is teleported, or the strangeness asymmetry of the two particles to which the entangled state is swapped. In this letter, we point out that one can also use measurement in the CP basis to verify quantum teleportation or entanglement swapping as well as to verify Einstein–Podolsky–Rosen correlations in general. The proposal and the verification methods can also be implemented in other neutral pseudoscalar mesons.

2 Entangled mesons

For any neutral pseudoscalar meson M^0 and its antiparticle $\bar{M}^0$, both with $J^P = 0^-$, an entangled state can be produced from a source with $J^{PC} = 1^{--}$. M^0 can be $K^0 = (d\bar{s})$, $B^0 = (d\bar{b})$, $\bar{B}_s^0 = (b\bar{s})$ or $\bar{D}^0 = (u\bar{c})$. $|M^0\rangle$ and $|\bar{M}^0\rangle$ are eigenstates of parity P with eigenvalues -1 and also eigenstates of its characteristic flavor $\mathcal{F}$ with eigenvalues 1 and -1, respectively. $\mathcal{F}$ is strangeness for kaons, beauty for B-mesons and charm for D-mesons, respectively. For B_s-mesons, $\mathcal{F}$ can be chosen to be strangeness or beauty (with minus sign). We have $C|M^0\rangle = -|\bar{M}^0\rangle$ and $C|\bar{M}^0\rangle = -|M^0\rangle$. Thus the eigenstates of CP are

$$|M_+\rangle = \frac{1}{\sqrt{2}}(|M^0\rangle + |\bar{M}^0\rangle)\,,$$
$$|M_-\rangle = \frac{1}{\sqrt{2}}(|M^0\rangle - |\bar{M}^0\rangle)\,,$$

with eigenvalues $+1$ and -1, respectively. Under weak interactions, the lifetime–mass eigenstates are $|M_{\mathrm{SL}}\rangle$ and $|M_{\mathrm{LH}}\rangle$, with eigenvalues $\lambda_{\mathrm{SL}} = m_{\mathrm{SL}} - \mathrm{i}\Gamma_{\mathrm{SL}}/2$ and $\lambda_{\mathrm{LH}} = m_{\mathrm{LH}} - \mathrm{i}\Gamma_{\mathrm{LH}}/2$, where the subscript SL means short lifetime and light mass, while LH means long lifetime and heavy mass. Usually, the first subscript is used for kaons while the second subscript is used for B-mesons, as kaons differ mainly in lifetimes while B-mesons differ mainly in masses. Here we write both subscripts for a general discussion. In terms of the proper time τ, the weak decay is described as $|M_{\mathrm{SL}}(\tau)\rangle = \mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{SL}}\tau}|M_{\mathrm{SL}}\rangle$ and $|M_{\mathrm{LH}}(\tau)\rangle = \mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{LH}}\tau}|M_{\mathrm{LH}}\rangle$. $|M_{\mathrm{SL}}\rangle$ and $|M_{\mathrm{LH}}\rangle$ are related to CP and strangeness eigenstates as

$$|M_{\mathrm{SL}}\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|M_+\rangle + \epsilon|M_-\rangle)$$
$$= \frac{1}{\sqrt{|p|^2+|q|^2}}(p|M^0\rangle + q|\bar{M}^0\rangle)\,,$$

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$$
\begin{aligned}
|M_{\mathrm{LH}}\rangle &= \frac{1}{\sqrt{1+|\epsilon|^2}}(|M_-\rangle+\epsilon|M_+\rangle) \\
&= \frac{1}{\sqrt{|p|^2+|q|^2}}(p|M^0\rangle-q|\bar{M}^0\rangle)\,, \qquad (1)
\end{aligned}
$$

where ϵ is the parameter characterizing CP violation, $p=1+\epsilon$ and $q=1-\epsilon$. The magnitude of ϵ is of the order of 10^{-3}.

The entangled state with $J^{PC}=1^{--}$, written in terms of the three bases respectively, is

$$
\begin{aligned}
|\Psi_-\rangle &= \frac{1}{\sqrt{2}}(|M^0\rangle|\bar{M}^0\rangle-|\bar{M}^0\rangle|M^0\rangle) \qquad (2)\\
&= \frac{1}{\sqrt{2}}(|M_-\rangle|M_+\rangle-|M_+\rangle|M_-\rangle) \qquad (3)\\
&= \frac{r}{\sqrt{2}}(|M_{\mathrm{LH}}\rangle|M_{\mathrm{SL}}\rangle-|M_{\mathrm{SL}}\rangle|M_{\mathrm{LH}}\rangle)\,, \qquad (4)
\end{aligned}
$$

where $r=(|p|^2+|q|^2)/2pq=(1+|\epsilon|^2)/(1-\epsilon^2)$.

It is remarkable that in the CP basis, $|\Psi_-\rangle$ is also a singlet strictly, and there is no dependence on the CP violation parameter, which only appears in the expression in the lifetime–mass basis. It should be noted that $|M_+\rangle$ and $|M_-\rangle$ are orthogonal to each other and are exactly distinguished by different values of CP, while $|M_{\mathrm{SL}}\rangle$ and $|M_{\mathrm{LH}}\rangle$ are not orthogonal to each other as a consequence of CP violation. This makes it advantageous to use the CP basis, rather than the weak basis, to do measurements.

In actual experiments, identification of $|M_+\rangle$ and $|M_-\rangle$ is made by the modes of nonleptonic decays, i.e. a M_+ decays to 2π, while a M_- decays to 3π. On the other hand, although the physical states in free propagation are $|M_{\mathrm{SL}}\rangle$ and $|M_{\mathrm{LH}}\rangle$, they cannot be exactly identified due to CP violation. In the so-called passive measurement of lifetime, one identifies the mesons that decay between τ and $\tau+\Delta\tau$ as $|M_{\mathrm{SL}}\rangle$ and those decaying later as $|M_{\mathrm{LH}}\rangle$, by choosing the appropriate τ and $\Delta\tau$ [1, 2]. There is always a nonzero misidentification probability. Also, it relies on the significant difference between Γ_{SL} and Γ_{LH} and hence does not apply to other cases such as B-mesons. Furthermore, one has to exclude the semileptonic decays, which are in the flavor basis. To do this, one has to look into the details of the decays. But the nonleptonic decays are just in the CP basis. Therefore we would rather abandon the identification in the lifetime–mass basis.

3 High energy quantum teleportation

In the following we first review the proposal of high energy teleportation and entanglement swapping. For the general case of M-mesons, one can extend the calculation results in [10] by replacing Γ_{S} and Γ_{L} with $i\lambda_{\mathrm{SL}}$ and $i\lambda_{\mathrm{LH}}$, respectively, as the mass difference for kaons has been neglected there.

Suppose two mesons labeled by a and b are produced in state $|\Psi_-\rangle$ at time $t=0$, in the laboratory frame that coincides with the center of mass frame. At time t, the state becomes

$$
|\Psi_{ab}(t)\rangle = M(t)|\Psi_-\rangle_{ab}\,, \qquad (5)
$$

where $M(t)=\exp[-\mathrm{i}(\lambda_{\mathrm{SL}}+\lambda_{\mathrm{LH}})\gamma_b^{-1}t]$. γ_i is the Lorentz factor $1/\sqrt{1-v_i^2}$ for particle i with velocity v_i. It has been assumed that $\gamma_a=\gamma_b$. A third kaon c is generated at time t_z, with $|\Psi_c(t_z)\rangle=\alpha|M^0\rangle_c+\beta|\bar{M}^0\rangle_c$; thus,

$$
\begin{aligned}
|\Psi_c(t)\rangle &= C(t)|M_{\mathrm{SL}}\rangle_c+B(t)|M_{\mathrm{LH}}\rangle_c \qquad (6)\\
&= D(t)|M_+\rangle_c+E(t)|M_-\rangle_c \qquad (7)\\
&= F(t)|M^0\rangle_c+G(t)|\bar{M}^0\rangle_c\,, \qquad (8)
\end{aligned}
$$

where

$$
\begin{aligned}
C(t) &= \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2}}\left(\frac{\alpha}{1+\epsilon}+\frac{\beta}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{SL}}\gamma_c^{-1}(t-t_z)}\,,\\
B(t) &= \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2}}\left(\frac{\alpha}{1+\epsilon}-\frac{\beta}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{LH}}\gamma_c^{-1}(t-t_z)}\,,\\
D(t) &= \frac{1}{\sqrt{2}}\left(\frac{\alpha}{1+\epsilon}+\frac{\beta}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{SL}}\gamma_c^{-1}(t-t_z)}\\
&\quad+\frac{\epsilon}{\sqrt{2}}\left(\frac{\alpha}{1+\epsilon}-\frac{\beta}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{LH}}\gamma_c^{-1}(t-t_z)}\,,\\
E(t) &= \frac{\epsilon}{\sqrt{2}}\left(\frac{\alpha}{1+\epsilon}+\frac{\beta}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{SL}}\gamma_c^{-1}(t-t_z)}\\
&\quad+\frac{1}{\sqrt{2}}\left(\frac{\alpha}{1+\epsilon}-\frac{\beta}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{LH}}\gamma_c^{-1}(t-t_z)}\,,\\
F(t) &= \frac{1}{2}\left[\left(\alpha+\beta\frac{1+\epsilon}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{SL}}\gamma_c^{-1}(t-t_z)}\right.\\
&\quad\left.+\left(\alpha-\beta\frac{1+\epsilon}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{LH}}\gamma_c^{-1}(t-t_z)}\right]\,,\\
G(t) &= \frac{1}{2}\left[\left(\alpha\frac{1-\epsilon}{1+\epsilon}+\beta\right)\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{SL}}\gamma_c^{-1}(t-t_z)}\right.\\
&\quad\left.-\left(\alpha\frac{1-\epsilon}{1+\epsilon}-\beta\right)\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{LH}}\gamma_c^{-1}(t-t_z)}\right]\,.
\end{aligned}
$$

The state of the three particles is thus

$$
|\Psi_{cab}(t)\rangle = |\Psi_c(t)\rangle\otimes|\Psi_{ab}(t)\rangle\,. \qquad (9)
$$

We let a and c fly in opposite directions and towards each other; hence, they collide at a certain position x at a certain time t_x. The collision can be represented as a unitary transformation $\mathcal{S}$ on c–a, in a negligible time duration δ much shorter than the lifetimes of weak decay. After the c–a collision, the state of the three kaons can be written as

$$
\begin{aligned}
|\Psi_{cab}(t_x+\delta)\rangle &= \frac{M(t_x)}{2}\Big\{\sqrt{2}F(t_x)\mathcal{S}|\phi_1\rangle_{ca}|\bar{M}^0\rangle_b\\
&\quad-\sqrt{2}G(t_x)\mathcal{S}|\phi_2\rangle_{ca}|M^0\rangle_b\\
&\quad-\mathcal{S}|\phi_3\rangle_{ca}[F(t_x)|M^0\rangle_b-G(t_x)|\bar{M}^0\rangle_b]\\
&\quad-\mathcal{S}|\phi_4\rangle_{ca}[F(t_x)|M^0\rangle_b+G(t_x)|\bar{M}^0\rangle_b]\Big\}\,, \qquad (10)
\end{aligned}
$$

where the $|\phi_i\rangle$ are eigenstates of parity P, strangeness S and isospin I: $|\phi_1\rangle\equiv|M^0M^0\rangle$ with $P=1$, $S=2$ and

$I=1$; $|\phi_2\rangle \equiv |\bar{M}^0\bar{M}^0\rangle$ with $P=1, S=-2$ and $I=1$; $|\phi_3\rangle \equiv |\Psi_+\rangle \equiv \frac{1}{\sqrt{2}}(|M^0\rangle|\bar{M}^0\rangle + |\bar{M}^0\rangle|M^0\rangle)$, with $P=1$, $S=0$ and $I=1$. Furthermore, $|\phi_4\rangle \equiv |\Psi_-\rangle$ with $P=-1$, $S=0$ and $I=0$. $\mathcal{S}|\phi_i\rangle$ is also an eigenstate of S, P and I with the same eigenvalue as $|\phi_i\rangle$, because $\mathcal{S}$ conserves S, P and I, as governed by the strong interaction.

The outcomes of the c–a collision are detected by using interaction with nuclear matter. Hence the state is projected in the basis $\{\mathcal{S}|\phi_i\rangle\}$. Conditioned on this projection, b is known to be, correspondingly, in one of the four states

$$|\bar{M}^0\rangle_b\,, \quad |M^0\rangle_b\,,$$
$$\frac{F(t_x)|M^0\rangle_b - G(t_x)|\bar{M}^0\rangle_b}{\sqrt{|F(t_x)|^2+|G(t_x)|^2}}\,,$$
$$\frac{F(t_x)|M^0\rangle_b + G(t_x)|\bar{M}^0\rangle_b}{\sqrt{|F(t_x)|^2+|G(t_x)|^2}}\,.$$

If and only if the outgoing particles of the c–a collision are detected to be with $P=-1$, $S=0$ and $I=0$, then is the b particle retained and known to be in state $[F(t_x)|M^0\rangle_b + G(t_x)|\bar{M}^0\rangle_b]/\sqrt{|F(t_x)|^2+|G(t_x)|^2}$, which is just the state of c before collision. This completes the teleportation from c to b. The four possible states of b at $t_x+\delta$, after knowing the four possible outcomes of the c–a collision, can be verified by measuring the flavor ratio of b, as suggested in [10].

Now we review entanglement swapping. In addition to $|\Psi_-\rangle_{ab}$ generated at time $t=0$, another meson pair d and c is generated as $|\Psi_-\rangle_{dc}$ at t_z. Consequently,

$$|\Psi_{dcab}(t)\rangle = M'(t-t_z)M(t)|\Psi_-\rangle_{dc}|\Psi_-\rangle_{ab}\,, \tag{11}$$

where $M'(t-t_z) = \exp[-\mathrm{i}(\lambda_{\mathrm{SL}}+\lambda_{\mathrm{LH}})\gamma_d^{-1}(t-t_z)]$, supposing $\gamma_c=\gamma_d$. Let c and a fly towards each other to collide at a certain time t_x. Within a negligible time interval δ, the collision brings about a unitary transformation $\mathcal{S}$ on c–a; therefore

$$\begin{aligned}|\Psi_{dcab}(t_x+\delta)\rangle = \frac{M'(t_x-t_z)M(t_x)}{2}\big(&\mathcal{S}|\Psi_+\rangle_{ca}|\Psi_+\rangle_{db}\\ &-\mathcal{S}|\Psi_-\rangle_{ca}|\Psi_-\rangle_{db}\\ &-\mathcal{S}|M^0M^0\rangle_{ca}|\bar{M}^0\bar{M}^0\rangle_{db}\\ &-\mathcal{S}|\bar{M}^0\bar{M}^0\rangle_{ca}|M^0M^0\rangle_{db}\big)\,,\end{aligned} \tag{12}$$

where

$$|\Psi_+\rangle_{ca} \equiv \frac{1}{\sqrt{2}}\left(|M^0\rangle_c|\bar{M}^0\rangle_a + |\bar{M}^0\rangle_c|M^0\rangle_a\right) \tag{13}$$
$$= \frac{r}{\sqrt{2}}\left(|M_{\mathrm{SL}}\rangle_c|M_{\mathrm{SL}}\rangle_a - |M_{\mathrm{LH}}\rangle_c|M_{\mathrm{LH}}\rangle_a\right). \tag{14}$$

Then, in measuring parity P, strangeness S and isospin I of the outgoing particles from the c–a collision, c and a are projected to one of the four states $\mathcal{S}|\Psi_+\rangle_{ca}$, $\mathcal{S}|\Psi_-\rangle_{ca}$, $\mathcal{S}|M^0M^0\rangle_{ca}$ and $\mathcal{S}|\bar{M}^0\bar{M}^0\rangle_{ca}$. Correspondingly, d and b are projected to $|\Psi_+\rangle_{db}$, $|\Psi_-\rangle_{db}$, $|\bar{M}^0\bar{M}^0\rangle_{db}$ and $|M^0M^0\rangle_{db}$, respectively. Accordingly one chooses to retain or abandon the b particle. The success of entanglement swapping can be verified by measuring the flavor asymmetry between the d and b particles, as suggested in [10].

4 *CP* measurement

4.1 Teleportation

Now we propose that the effect of teleportation and entanglement swapping can both be verified by measurement in the CP basis. First we discuss the verification of the effect of teleportation. In the teleportation scheme outlined above, the state of b, after knowing the outcomes of the c–a collision, can be verified in the CP basis. One measures the ratio η between the probabilities for b to be in $|M_+\rangle$ and in $|M_-\rangle$. For

$$|\Psi(t\geq t_x+\delta)\rangle_b = u_+(t)|M_+\rangle_b + u_-(t)|M_-\rangle_b\,,$$
$$\eta(t)\equiv\left|\frac{u_+(t)}{u_-(t)}\right|^2.$$

Many runs of the procedure, or many copies of b particles in a beam, are needed to determine this quantity.

If irrespective of the outcome of the c–a collision, b particles in different runs of the experiment are all considered in measuring $\eta(t)$, then $\eta(t)$ should be calculated by using $|\Psi_{cab}(t)\rangle$, given in (9). Because b is maximally entangled with a, it can be found that $\eta_b(t)=1$. In contrast, if only b particles in those runs of the experiment with a certain projection result of c–a are considered in measuring $\eta(t)$, then $\eta(t)$ is calculated by using the corresponding projected state of b, as seen in (10). Denote the state of b following the projection as $\alpha'|M^0\rangle + \beta'|\bar{M}^0\rangle$. Its subsequent evolution in the CP basis is then similar to (7), with t_z substituted for by $t_x+\delta$, γ_c by γ_b, α by α' and β by β'. It can be found that

$$\eta_b(t) = \left|\frac{\left(\frac{\alpha'}{1+\epsilon}+\frac{\beta'}{1-\epsilon}\right)+\epsilon\left(\frac{\alpha'}{1+\epsilon}-\frac{\beta'}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\Delta\lambda\tau}}{\epsilon\left(\frac{\alpha'}{1+\epsilon}+\frac{\beta'}{1-\epsilon}\right)+\left(\frac{\alpha'}{1+\epsilon}-\frac{\beta'}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\Delta\lambda\tau}}\right|^2,$$

where $\tau=\gamma_b^{-1}(t-t_x-\delta)$ and $\Delta\lambda=\lambda_{\mathrm{LH}}-\lambda_{\mathrm{SL}}=\Delta m - i\Delta\Gamma$, with $\Delta m = m_{\mathrm{LH}}-m_{\mathrm{SL}}$ and $\Delta\Gamma=\Gamma_{\mathrm{LH}}-\Gamma_{\mathrm{SL}}/2$.

For each of the four projection cases, $\eta_b(t)$ is very different from 1. If c–a projects to $\mathcal{S}|M^0M^0\rangle$, then

$$\eta_b = \left|\frac{1-\epsilon\mathrm{e}^{-\mathrm{i}\Delta\lambda\tau}}{\epsilon-\mathrm{e}^{-\mathrm{i}\Delta\lambda\tau}}\right|^2.$$

If c–a projects to $\mathcal{S}|\bar{M}^0\bar{M}^0\rangle$, then

$$\eta_b = \left|\frac{1+\epsilon\mathrm{e}^{-\mathrm{i}\Delta\lambda\tau}}{\epsilon+\mathrm{e}^{-\mathrm{i}\Delta\lambda\tau}}\right|^2.$$

If c–a projects to $\mathcal{S}|\Psi_+\rangle$, then

$$\eta_b(t) = \left|\frac{\left(\frac{F(t_x)}{1+\epsilon}-\frac{G(t_x)}{1-\epsilon}\right)+\epsilon\left(\frac{F(t_x)}{1+\epsilon}+\frac{G(t_x)}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\Delta\lambda\tau}}{\epsilon\left(\frac{F(t_x)}{1+\epsilon}-\frac{G(t_x)}{1-\epsilon}\right)+\left(\frac{F(t_x)}{1+\epsilon}+\frac{G(t_x)}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\Delta\lambda\tau}}\right|^2.$$

If c–a projects to $\mathcal{S}|\Psi_-\rangle$, i.e. the teleportation is successful, then

$$\eta_b(t) = \left|\frac{\left(\frac{F(t_x)}{1+\epsilon}+\frac{G(t_x)}{1-\epsilon}\right)+\epsilon\left(\frac{F(t_x)}{1+\epsilon}-\frac{G(t_x)}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\Delta\lambda\tau}}{\epsilon\left(\frac{F(t_x)}{1+\epsilon}+\frac{G(t_x)}{1-\epsilon}\right)+\left(\frac{F(t_x)}{1+\epsilon}-\frac{G(t_x)}{1-\epsilon}\right)\mathrm{e}^{-\mathrm{i}\Delta\lambda\tau}}\right|^2.$$

4.2 Entanglement swapping

Now we turn to entanglement swapping, which can also be verified by measurement in the CP basis. One can measure CP asymmetry between b and d, defined as

$$A_{cp}(t) = \frac{p_{d,cp}(t) - p_{s,cp}(t)}{p_{d,cp}(t) + p_{s,cp}(t)},$$

where $p_{d,cp}(t)$ and $p_{s,cp}(t)$ are, respectively, the probabilities for b and d to have different and the same values of CP [5]. Many runs of the experiment, or many copies of the particles in a beam, are needed to experimentally determine $A_{cp}(t)$. If all the d–b pairs in different runs are considered, irrespective of the projection results of c–a, it can be found that $A_{cp}(t) = 0$, as calculated from $|\Psi_{dcab}(t)\rangle$, by (4), (7) and (11). In contrast, if only those d–b pairs corresponding to a certain projection result of the c–a collision are considered, then $A_{cp}(t)$ is calculated by using the corresponding projected state of d and b, as determined from $|\Psi_{dcab}(t_x + \delta)\rangle$ given in (12). In the following, we give calculations for the four cases of projection.

Case 1. If at $t = t_x + \delta$, c and a are projected to $\mathcal{S}|\Psi_+\rangle_{ca}$, then

$$\begin{aligned}
&\Psi(t \geq t_x + \delta)\rangle_{db} \\
&= \frac{r}{\sqrt{2}} \Big[\mathrm{e}^{-\mathrm{i}(\lambda_{\mathrm{SL}}\tau_d + \lambda_{\mathrm{SL}}\tau_b)} |M_{\mathrm{SL}}\rangle_d |M_{\mathrm{SL}}\rangle_b \\
&\quad - \mathrm{e}^{-\mathrm{i}(\lambda_{\mathrm{LH}}\tau_d + \lambda_{\mathrm{LH}}\tau_b)} |M_{\mathrm{LH}}\rangle_d |M_{\mathrm{LH}}\rangle_b \Big] \\
&= \frac{1}{\sqrt{2}(1-\epsilon^2)} \left[\epsilon(f_1 - f_2)(|M_- M_+\rangle + |M_+ M_-\rangle) \right. \\
&\quad \left. + (\epsilon^2 f_1 - f_2)|M_- M_-\rangle + (f_1 - \epsilon^2 f_2)|M_+ M_+\rangle \right],
\end{aligned}$$

where $\tau_d = \gamma_d^{-1}(t - t_x - \delta)$, $\tau_b = \gamma_b^{-1}(t - t_x - \delta)$, $f_1 = \mathrm{e}^{-\mathrm{i}(\lambda_{\mathrm{SL}}\tau_d + \lambda_{\mathrm{SL}}\tau_b)}$, $f_2 = \mathrm{e}^{-\mathrm{i}(\lambda_{\mathrm{LH}}\tau_d + \lambda_{\mathrm{LH}}\tau_b)}$. As d and b originate from different sources, γ_d and γ_b may be different. Consequently

$$\begin{aligned}
&A_{cp}(t \geq t_x + \delta) \\
&= \Big\{ -(1 + a_{\epsilon 0})^2 (1 + \mathrm{e}^{-\Delta\Gamma(\tau_d + \tau_b)}) \\
&\quad - 2a_{\epsilon+}^2 \mathrm{e}^{-\frac{\Delta\Gamma}{2}(\tau_d + \tau_b)} \cos\Delta m(\tau_d + \tau_b) \Big\} \\
&\quad \times \frac{1}{1 + \mathrm{e}^{-\Delta\Gamma(\tau_d + \tau_b)} - 2a_\epsilon^2 \mathrm{e}^{-\frac{\Delta\Gamma}{2}(\tau_d + \tau_b)} \cos\Delta m(\tau_d + \tau_b)},
\end{aligned}$$

where we have used rephase-invariant CP-violating observables [11]

$$a_\epsilon = \frac{2\mathrm{Re}\epsilon}{1 + |\epsilon|^2}, \quad a_{\epsilon+} = \frac{2\mathrm{Im}\epsilon}{1 + |\epsilon|^2},$$

which characterize indirect CP violation and mixing-induced CP violation, respectively, and the third quantity $a_{\epsilon 0} = -2|\epsilon|^2/(1 + |\epsilon|^2)$, which is related to a_ϵ and $a_{\epsilon+}$ through $(1 + a_{\epsilon 0})^2 + a_{\epsilon+}^2 = 1 - a_\epsilon^2$ [11]. As CP violation is small in the neutral meson system, $a_\epsilon \ll 1$, $a_{\epsilon+} \ll 1$, and thus $a_{\epsilon 0} \ll 1$. Therefore A_{cp} is close to -1.

Case 2. If at $t = t_x + \delta$, c and a are projected to $\mathcal{S}|\Psi_-\rangle_{ca}$, i.e. the entanglement swapping is successful, then for $t \geq t_x + \delta$,

$$\begin{aligned}
|\Psi(t \geq t_x + \delta)\rangle_{db} &= \frac{r}{\sqrt{2}} \Big[\mathrm{e}^{-\mathrm{i}(\lambda_{\mathrm{LH}}\tau_d + \lambda_{\mathrm{SL}}\tau_b)} |M_{\mathrm{LH}}\rangle_d |M_{\mathrm{SL}}\rangle_b \\
&\quad - \mathrm{e}^{-\mathrm{i}(\lambda_{\mathrm{SL}}\tau_d + \lambda_{\mathrm{LH}}\tau_b)} |M_{\mathrm{SL}}\rangle_d |M_{\mathrm{LH}}\rangle_b \Big] \\
&= \frac{1}{\sqrt{2}(1-\epsilon^2)} \left[\left(g_1 - \epsilon^2 g_2\right) |M_- M_+\rangle \right. \\
&\quad + \epsilon(g_1 - g_2)(|M_- M_-\rangle + |M_+ M_+\rangle) \\
&\quad \left. + \left(\epsilon^2 g_1 - g_2\right) |M_+ M_-\rangle \right],
\end{aligned}$$

where $g_1 = \mathrm{e}^{-\mathrm{i}(\lambda_{\mathrm{LH}}\tau_d + \lambda_{\mathrm{SL}}\tau_b)}$, $g_2 = \mathrm{e}^{-\mathrm{i}(\lambda_{\mathrm{SL}}\tau_d + \lambda_{\mathrm{LH}}\tau_b)}$. Consequently

$$\begin{aligned}
&A_{cp}(t \geq t_x + \delta) \\
&= \Big\{ (1 + a_{\epsilon 0})^2 \left(1 + \mathrm{e}^{-\Delta\Gamma(\tau_d - \tau_b)}\right) \\
&\quad + 2a_{\epsilon+}^2 \mathrm{e}^{-\frac{\Delta\Gamma}{2}(\tau_d - \tau_b)} \cos\Delta m(\tau_d - \tau_b) \Big\} \\
&\quad \times \frac{1}{1 + \mathrm{e}^{-\Delta\Gamma(\tau_d - \tau_b)} - 2a_\epsilon^2 \mathrm{e}^{-\frac{\Delta\Gamma}{2}(\tau_d - \tau_b)} \cos\Delta m(\tau_d - \tau_b)},
\end{aligned}$$

which is close to 1.

Case 3. If at $t = t_x + \delta$, c and a are projected to $\mathcal{S}|M^0 M^0\rangle_{ca}$, then for $t \geq t_x + \delta$,

$$\begin{aligned}
&|\Psi(t \geq t_x + \delta)\rangle_{db} \\
&= \frac{1 + |\epsilon|^2}{2(1-\epsilon)^2} [f_1 |M_{\mathrm{SL}}\rangle_d |M_{\mathrm{SL}}\rangle_b \\
&\quad - g_2 |M_{\mathrm{SL}}\rangle_d |M_{\mathrm{LH}}\rangle_b - g_1 |M_{\mathrm{LH}}\rangle_d |M_{\mathrm{SL}}\rangle_b \\
&\quad + f_2 |M_{\mathrm{LH}}\rangle_d |M_{\mathrm{LH}}\rangle_b] \\
&= \frac{1}{2(1-\epsilon)^2} \left[\left(f_1 - \epsilon g_2 - \epsilon g_1 + \epsilon^2 f_2\right) |M_+ M_+\rangle \right. \\
&\quad + \left(\epsilon f_1 - g_2 - \epsilon^2 g_1 + \epsilon f_2\right) |M_+ M_-\rangle \\
&\quad + \left(\epsilon f_1 - \epsilon^2 g_2 - g_1 + \epsilon f_2\right) |M_- M_+\rangle \\
&\quad \left. + \left(\epsilon^2 f_1 - \epsilon g_2 - \epsilon g_1 + f_2\right) |M_- M_-\rangle \right].
\end{aligned}$$

Consequently,

$$\begin{aligned}
&A_{cp}(t \geq t_x + \delta) \\
&= \Big\{ 2a_{\epsilon+} \left[(1 - |\epsilon|^2) \mathrm{e}^{-2\Gamma_{\mathrm{SL}}\tau_d} - \mathrm{e}^{-2\Gamma_{\mathrm{LH}}\tau_d} \right] \\
&\quad \times \mathrm{e}^{-(\Gamma_{\mathrm{LH}} + \Gamma_{\mathrm{SL}})\tau_b} \sin\Delta m\tau_b \\
&\quad + 2a_{\epsilon+} \left[\mathrm{e}^{-2\Gamma_{\mathrm{SL}}\tau_b} - (1 - |\epsilon|^2) \mathrm{e}^{-2\Gamma_{\mathrm{LH}}\tau_b} \right] \\
&\quad \times \mathrm{e}^{-(\Gamma_{\mathrm{LH}} + \Gamma_{\mathrm{SL}})\tau_d} \sin\Delta m\tau_d \\
&\quad + 2a_\epsilon |\epsilon|^2 \Big(\mathrm{e}^{-[2\Gamma_{\mathrm{SL}}\tau_b + (\Gamma_{\mathrm{LH}} + \Gamma_{\mathrm{SL}})\tau_d]} \cos\Delta m\tau_d \\
&\quad + \mathrm{e}^{-[2\Gamma_{\mathrm{LH}}\tau_d + (\Gamma_{\mathrm{LH}} + \Gamma_{\mathrm{SL}})\tau_b]} \cos\Delta m\tau_b \Big) \\
&\quad + 4a_{\epsilon+}^2 (1 + |\epsilon|^2) \mathrm{e}^{-(\Gamma_{\mathrm{LH}} + \Gamma_{\mathrm{SL}})(\tau_d + \tau_b)} \cos\Delta m\tau_d \cos\Delta m\tau_b \Big\}
\end{aligned}$$

$$
\begin{aligned}
&\Big/ \Big\{ (1+|\epsilon|^2) \Big(\mathrm{e}^{-\Gamma_{\mathrm{SL}}(\tau_d+\tau_b)} + \mathrm{e}^{-(\Gamma_{\mathrm{SL}}\tau_d+\Gamma_{\mathrm{LH}}\tau_b)} \\
&+\mathrm{e}^{-\Gamma_{\mathrm{LH}}(\tau_d+\tau_b)}\mathrm{e}^{-(\Gamma_{\mathrm{LH}}\tau_d+\Gamma_{\mathrm{SL}}\tau_b)} \Big) \\
&-2a_\epsilon \left[(1+|\epsilon|^2)\mathrm{e}^{-2\Gamma_{\mathrm{SL}}\tau_d} + \mathrm{e}^{-2\Gamma_{\mathrm{LH}}\tau_d}\right] \\
&\times \mathrm{e}^{-(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})\tau_b} \cos \Delta m \tau_b \\
&-2a_\epsilon \left[(1+|\epsilon|^2)\mathrm{e}^{-2\Gamma_{\mathrm{LH}}\tau_b} + \mathrm{e}^{-2\Gamma_{\mathrm{SL}}\tau_b}\right] \\
&\times \mathrm{e}^{-(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})\tau_d} \cos \Delta m \tau_d \\
&+2a_{\epsilon+}|\epsilon|^2 \Big(\mathrm{e}^{-[2\Gamma_{\mathrm{SL}}\tau_b+(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})\tau_d]} \sin \Delta m \tau_d \\
&-\mathrm{e}^{-[2\Gamma_{\mathrm{LH}}\tau_d+(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})\tau_b]} \sin \Delta m \tau_b \Big) \\
&+4a_\epsilon^2(1+|\epsilon|^2)\mathrm{e}^{-(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})(\tau_d+\tau_b)} \cos \Delta m \tau_d \cos \Delta m \tau_b \Big\},
\end{aligned}
$$

which has been written in terms of the rephase-invariant quantities a_ϵ and $a_{\epsilon+}$, as well as $|\epsilon|^2$, which can be substituted for by $-a_{\epsilon 0}/(2+a_{\epsilon 0})$. In this case, A_{cp} is close to 0.

Case 4. If at $t=t_x+\delta$, c and a are projected to $\mathcal{S}|\bar{M}^0\bar{M}^0\rangle_{ca}$, then for $t \geq t_x+\delta$

$$
\begin{aligned}
&|\Psi(t \geq t_x+\delta)\rangle_{db} \\
&\quad = \frac{1+|\epsilon|^2}{2(1+\epsilon)^2} \left[f_1|M_{\mathrm{SL}}\rangle_d|M_{\mathrm{SL}}\rangle_b + g_2|M_{\mathrm{SL}}\rangle_d|M_{\mathrm{LH}}\rangle_b \right. \\
&\qquad \left. +g_1|M_{\mathrm{LH}}\rangle_d|M_{\mathrm{SL}}\rangle_b + f_2|M_{\mathrm{LH}}\rangle_d|M_{\mathrm{LH}}\rangle_b \right] \\
&\quad = \frac{1}{2(1+\epsilon)^2} \left[\left(f_1+\epsilon g_2+\epsilon g_1+\epsilon^2 f_2\right)|M_+M_+\rangle \right. \\
&\qquad + \left(\epsilon f_1+g_2+\epsilon^2 g_1+\epsilon f_2\right)|M_+M_-\rangle \\
&\qquad + \left(\epsilon f_1+\epsilon^2 g_2+g_1+\epsilon f_2\right)|M_-M_+\rangle \\
&\qquad \left. + \left(\epsilon^2 f_1+\epsilon g_2+\epsilon g_1+f_2\right)|M_-M_-\rangle \right].
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
&A_{cp}(t \geq t_x+\delta) \\
&= \Big\{ 2a_{\epsilon+}(1-|\epsilon|^2) \\
&\times \Big[\left(\mathrm{e}^{-2\Gamma_{\mathrm{LH}}\tau_b} - \mathrm{e}^{-2\Gamma_{\mathrm{SL}}\tau_b}\right) \mathrm{e}^{-(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})\tau_d} \sin \Delta m \tau_d \\
&+ \left(\mathrm{e}^{-2\Gamma_{\mathrm{LH}}\tau_d} - \mathrm{e}^{-2\Gamma_{\mathrm{SL}}\tau_d}\right) \mathrm{e}^{-(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})\tau_b} \sin \Delta m \tau_b \Big] \\
&+4a_{\epsilon+}^2(1+|\epsilon|^2)\mathrm{e}^{-(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})(\tau_d+\tau_b)} \cos \Delta m \tau_d \cos \Delta m \tau_b \Big\} \\
&\Big/ \Big\{ (1+|\epsilon|^2) \\
&\times \Big(\mathrm{e}^{-\Gamma_{\mathrm{SL}}(\tau_d+\tau_b)} \\
&+\mathrm{e}^{-(\Gamma_{\mathrm{SL}}\tau_d+\Gamma_{\mathrm{LH}}\tau_b)} + \mathrm{e}^{-\Gamma_{\mathrm{LH}}(\tau_d+\tau_b)}\mathrm{e}^{-(\Gamma_{\mathrm{LH}}\tau_d+\Gamma_{\mathrm{SL}}\tau_b)} \Big) \\
&+2a_\epsilon(1+|\epsilon|^2) \\
&\times \Big[\left(\mathrm{e}^{-2\Gamma_{\mathrm{SL}}\tau_d} + \mathrm{e}^{-2\Gamma_{\mathrm{LH}}\tau_d}\right) \mathrm{e}^{-(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})\tau_b} \cos \Delta m \tau_b \\
&+(\mathrm{e}^{-2\Gamma_{\mathrm{LH}}\tau_b} + \mathrm{e}^{-2\Gamma_{\mathrm{SL}}\tau_b})\mathrm{e}^{-(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})\tau_d} \cos \Delta m \tau_d \Big] \\
&+4a_\epsilon^2(1+|\epsilon|^2)\mathrm{e}^{-(\Gamma_{\mathrm{LH}}+\Gamma_{\mathrm{SL}})(\tau_d+\tau_b)} \cos \Delta m \tau_d \cos \Delta m \tau_b \Big\},
\end{aligned}
$$

which has been written in terms of the rephase-invariant quantities a_ϵ and $a_{\epsilon+}$, as well as $|\epsilon|^2$, which can be substituted for by $-a_{\epsilon 0}/(2+a_{\epsilon 0})$. In this case, A_{cp} is close to 0.

Therefore, no matter whether the two lifetimes are considerably different from each other, and no matter whether CP is violated, the success of the entanglement swapping can be clearly distinguished from the other three cases of projection, as well as from the case of no projection, by measuring the CP asymmetry. Obviously, CP asymmetry can also be similarly used in verifying the general teleportation, in which the teleported kaons are entangled arbitrarily with other particles in an unknown way [10].

Certainly, CP asymmetry can also be used for verifying the Einstein–Podolsky–Rosen state $|\Psi_-\rangle$ generated in e^+e^- or $p\bar{p}$ collisions, as in CPLEAR and BELLE, where flavor asymmetries were measured. However, it should be noted that these asymmetry quantities, including those measured in the CPLEAR and BELLE experiments, are by no means rigorous proofs of entanglement, because it is easy to construct separated states with the same asymmetry as an entangled state.

5 Summary

To summarize, we have described a scheme of using measurement in the CP basis to verify quantum teleportation or entanglement swapping in terms of neutral pseudoscalar mesons. This method has several advantages. It is rigorous, and it remains valid in the presence of CP violation. It also works efficiently, clearly distinguishing the success case in teleportation or entanglement swapping from other cases. Furthermore, this method is much simpler than the flavor measurement using the strong basis. The latter needs nuclear matter to interact with the particles to be detected, while in the present method, only the decay modes need to be determined. This aspect brings high energy quantum information manipulation in general, and quantum teleportation or entanglement swapping in particular, closer to actual experimental implementation, which, however, should still need to overcome other serious difficulties, e.g. precise control of the timing, careful determination of collision outcomes, etc. Finally, we emphasize that CP measurements can also be used in general tests of quantum mechanical effects in particle physics.

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